Chiquita Flux Calculation

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Original fit to the data (revised derivation)

The most basic fit one can use on the Chiquita data is a straight line in log space. To do this, we assume that the number flux (per eV) of cosmic rays follows the power law

$$J_0(E_0) = \frac{dN_0}{dE_0} = CE_0^{-\gamma}$$

where $\gamma_0 = 3$ and $\log C \sim 24.5$. Here E_0 is the actual energy, in eV, of the incoming particle (as opposed to the measured energy E).

For convenience in fitting the energy range measured by Chiquita, this function will be rewritten to be centered on $10^{17.8}$ eV.

$$\frac{dN_0}{dE_0} = C_{17.8} \left(\frac{E_0}{10^{17.8}} \right)^{-\gamma}.$$

The relationship between C and $C_{17.8}$ is $C = C_{17.8} \times 10^{17.8\gamma}$.

We are measuring the number flux per interval in observed energy, $\frac{dN}{dE}$. This is the convolution of the actual number flux with the function $R(E, E_0)$, which describes the distribution of reconstructed energies at each value of E_0.

$$J(E) = \frac{dN}{dE} = \int dE_0 A(E_0) R(E, E_0) \frac{dN_0}{dE_0}.$$

The function $A(E_0)$ is the acceptance of the array (the fraction of showers which pass all cuts on the data at a given energy).

The histogram of Chiquita data is binned in log space, with bin width $\log(E_{\text{upper}}) - \log(E_{\text{lower}}) = 0.2$. In terms of the central energy E_C of the bin, the bin width is

$$E_{\text{upper}} - E_{\text{lower}} = 10^{0.1} E_C - 10^{-0.1} E_C = 0.46 E_C.$$

Each data point in the histogram therefore contains a number of showers given by

$$N = \int_{10^{-0.1} E_C}^{10^{0.1} E_C} \frac{dN}{dE} dE.$$

To obtain the number of Chiquita showers in a given bin, we substitute the expression for $\frac{dN}{dE}$ above:

$$N(E_i) = \int_{\text{E}_\text{i} \text{ bin}} dE \int_{\text{all } \text{E}_0} dE_0 A(E_0) R(E, E_0) \frac{dN_0}{dE_0}$$

where E_i is the central energy (in log space) of the ith bin.

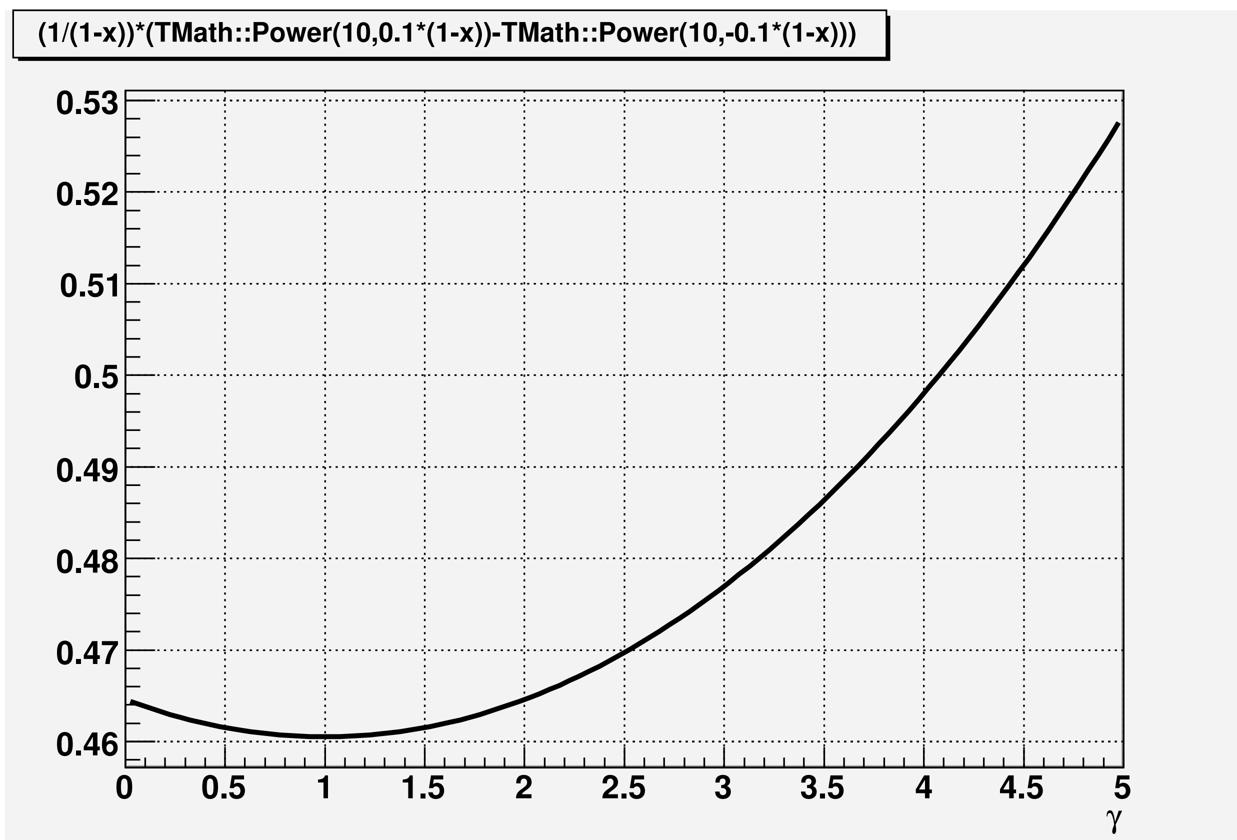
We have the function $R(E, E_0)$ in the form of a data table for a fixed set of values of E_0, so instead of a continuous integral, we need a sum. We make the approximation that the entire flux in the bin centered around E_0 is being transferred to the bin centered around the observed energy E with the efficiency $R(E, E_0)$.[1]

$$N(E_i) = \sum_{E_{0j}} A(E_{0j}) R(E_i, E_{0j}) \int_{\mathrm{E_{0j}\ bin}} dE_0 \frac{dN_0}{dE_0}$$

The integral of the flux over the E_{0j} bin is given by

$$\begin{aligned}
\int_{\mathrm{E_{0j}\ bin}} C E_0^{-\gamma} dE_0 &= \int_{10^{-0.1 E_{0j}}}^{10^{0.1 E_{0j}}} C E_0^{-\gamma} dE_0 \\
&= \left. \frac{C E_0^{-\gamma+1}}{-\gamma+1} \right|_{10^{-0.1 E_{0j}}}^{10^{0.1 E_{0j}}} \\
&= \frac{C}{-\gamma+1} \left[\left(10^{0.1}\right)^{-\gamma+1} - \left(10^{-0.1}\right)^{-\gamma+1} \right] E_{0j}^{-\gamma+1}
\end{aligned}$$

This function is flat at $\gamma = 1$. At values of γ close to 3, it can be reasonably approximated by the value at $\gamma = 3$.



[1] The data table of values for $R(E, E_0)$ was calculated using simulated showers with an E^{-3} distribution over the width of the bin. It is not the efficiency for moving showers from the central energy of one bin to the central energy of another bin, but rather an approximation to the fraction of showers in one bin which get moved to another bin due to error in the reconstructed energies. This is why substituting $R(E_i, E_{0j})$ for $R(E, E_0)$ takes care of the integral over the E_i bin. Note, however, that the farther γ is from 3, the more error is introduced through R. (This also applies to $A(E)$, which is tabulated from the same set of simulated showers.)

For $\gamma = 3$,

$$\int_{\mathrm{E_0\ bin}} C_0 E_0^{-\gamma} dE_0 = 0.477 C_0 E_0^{-\gamma+1}.$$

The function which should be used to fit the data[2] is thus

$$\begin{aligned} N(E_i) &\simeq \sum_{E_{0j}} 0.477 A(E_{0j}) R(E_i, E_{0j}) C E_{0j}^{-\gamma+1} \\ &= \sum_{E_{0j}} 0.477 A(E_{0j}) R(E_i, E_{0j}) C'_{17.8} \left(\frac{E_{0j}}{10^{17.8}}\right)^{-\gamma'} \end{aligned}$$

where $\gamma' = \gamma - 1$.

We want to do the fit to C and γ in log space, so this is rewritten as

$$N(E_i) = \sum_{E_{0j}} 0.477 A(E_{0j}) R(E_i, E_{0j}) 10^{\log C'_{17.8} - \gamma'(\log E_{0j} - 17.8)}.$$

The factor $\log C_{17.8}$ has been replaced by $\log C'_{17.8}$, where

$$\log C = \log C_{17.8} + 17.8\gamma = \log C'_{17.8} + 17.8\gamma'.$$

What we want to plot is

$$J \times E^3 = \frac{dN}{dE} E^3 = C E^{-\gamma} E^3.$$

This is calculated as

$$J \times E^3 = \frac{dN}{dE} E^3 = C E^{-\gamma} E^3 = C E^{-(\gamma-1)} E^{-1} E^3 = C'_{17.8} \left(\frac{E}{10^{17.8}}\right)^{-\gamma'} E^2.$$

[2]In the original derivation, I had $\left[10^{0.1} - 10^{-0.1}\right]$ where I should have had $\frac{1}{-\gamma+1}\left[\left(10^{0.1}\right)^{-\gamma+1} - \left(10^{-0.1}\right)^{-\gamma+1}\right]$. This gave a constant factor of 0.465, which is still close to the value of the second expression when $\gamma = 3$. Ideally the entire expression should be included when fitting for γ. Alternatively, one could iterate on the value.

Iterative fit method

In this method, we start by defining the observed flux $J(E) = \frac{dN}{dE}$ as before:

$$J(E) = \frac{dN}{dE} = \int dE_0 A(E_0) R(E, E_0) \frac{dN_0}{dE_0} = \int dE_0 A(E_0) R(E, E_0) J_0(E_0).$$

As shown in the previous section, a reasonable approximation to the number of showers in the ith energy bin is

$$N(E_i) \simeq \sum_{E_{0j}} 0.477 A(E_{0j}) R(E_i, E_{0j}) C E_{0j}^{-\gamma+1}$$

We would like to get rid of the sum in order to turn this into an expression for C. To do this, we want to define an effective aperture $A'(E)$ such that

$$N(E) = 0.477 E\, A'(E)\, J(E).$$

$A'(E)$ is calculated from simulations by taking the number of simulated showers which reconstruct in bin E_i over the number of simulated showers thrown in bin E_i. (This will not be the correct ratio if the showers were not thrown with the correct $E^{-\gamma}$ spectrum.) The number of showers which reconstruct in bin E_i is given by

$$N_{\text{sim}}(E_i) = \int_{\mathrm{E_i\ bin}} dE \int_{\mathrm{all\ E_0}} dE_0 A(E_0) R(E, E_0) J_{\text{sim}}(E_0).$$

The effective aperture is therefore given by

$$A'(E_i) = \frac{N_{\text{sim}}(E_i)}{\int_{\mathrm{E_i\ bin}} J_{\text{sim}}(E) dE}.$$

Similarly, the measured number of showers in each bin is

$$\begin{aligned} N(E_i) &= A'(E_i) \int_{\mathrm{E_i\ bin}} J_0(E) dE \\ &= A'(E_i) \int_{\mathrm{E_i\ bin}} C E^{-\gamma} dE \\ &\simeq 0.477 A'(E_i) C E_i^{-\gamma+1} \\ N(E_i) &= 0.477 E_i A'(E_i) J(E_i) \end{aligned}$$

Note that this is an expression which relates the total number of showers in the bin, $N(E_i)$, to the flux at a single point, $J(E = E_i)$. The aperture, $A'(E_i)$ is an average over the bin, and depends on the shape of the spectrum.

We have assumed that the flux $J(E)$ has the form $J(E) = CE^{-\gamma}$. The function we ultimately want to plot is $J(E)E^3 = CE^{3-\gamma}$. The set of points given by this function should be approximately constant:

$$C_{\text{data}}(E_i) \equiv J(E_i) E_i^3 = C E_i^{3-\gamma}$$

Substituting in the expression for $N(E_i)$ above,

$$C_{\text{data}}(E_i) = \frac{N(E_i) E_i^2}{0.477 A'(E_i)}.$$

We can use this expression to plot the points C_{data}, and then fit those points to the expression $C_{\text{data}}(E) = CE^{3-\gamma}$.

Call the best-fit values of these parameters C_2 and γ_2. We can use these values to generate a data set assuming an input spectrum of $J_2(E) = C_2 E^{-\gamma_2}$. (In other words, we work backward to see what the data set would look like if it were a perfect fit to the parameters C_2 and γ_2.) The generated data set is calculated from the function

$$N_{\rm data}(E_i) = 0.477 E_i\, A'(E_i)\, C_2 E_i^{-\gamma_2}.$$

In the code, this is actually calculated from the earlier expression

$$N_{\rm data}(E_i) = \sum_{E_{0j}} 0.477\, A(E_{0j}) R(E_i, E_{0j})\, C_2 E_{0j}^{-\gamma_2+1}.$$

As before, a new function $C'_{\rm data}(E)$ is defined by the points

$$C'_{\rm data}(E_i) = \frac{N_2(E_i) E_i^2}{0.477 A'(E_i)}.$$

Assuming no errors entered into the conversion from $C_2 E_i^{-\gamma_2}$ to $N_{\rm data}$ and back to $C'_{\rm data}$, the C' points would lie on the line given by the best-fit values to $C_{\rm data}$. (Error is introduced through the tabulated values of $A'(E)$, which were are based on simulations which assume an E^{-3} spectrum.)

The flux is now estimated as

$$J(E_i) = \frac{C_{\rm data}(E_i)}{C'_{\rm data}(E_i)} C_2 E_i^{-\gamma_2}.$$

Update

The iterative fit is now being done as follows:

We want to plot flux times E^3:

$$JE^3 = CE^{3-\gamma}$$

where we have assumed that $J = CE^{-\gamma}$.

We can extract C from the sum in the previous section:

$$C = \frac{N(E_i)}{\sum_{E_{0j}} 0.477\, A(E_{0j}) R(E_i, E_{0j})\, E_{0j}^{-\gamma+1}}$$

This gives:

$$JE^3 = \frac{N(E_i) E_i^{3-\gamma}}{\sum_{E_{0j}} 0.477\, A(E_{0j}) R(E_i, E_{0j})\, E_{0j}^{-\gamma+1}}$$

By assuming $\gamma = 3$, some factors of E disappear:

$$C_{\mathrm{data}}(E) = JE^3 = \frac{N(E_i)}{\sum_{E_{0j}} 0.477\, A(E_{0j}) R(E_i, E_{0j})\, E_{0j}^{-2}}$$

By fitting a line to this function, we get values for C and γ.

To perform one iteration on this result, we first calculate the number of showers per bin that would correspond to the fitted values of C and γ:

$$N_{\mathrm{data}}(E_i) = \sum_{E_{0j}} 0.477\, A(E_{0j}) R(E_i, E_{0j})\, C_2 E_{0j}^{-\gamma_2+1}.$$

This generated data set is used to calculate a new histogram of JE^3:

$$C'_{\mathrm{data}}(E) = JE^3 = \frac{N_{\mathrm{data}}(E_i)}{\sum_{E_{0j}} 0.477\, A(E_{0j}) R(E_i, E_{0j})\, E_{0j}^{-2}}$$

As before, the flux is now estimated as

$$J(E_i) = \frac{C_{\mathrm{data}}(E_i)}{C'_{\mathrm{data}}(E_i)} C_2 E_i^{-\gamma_2}.$$